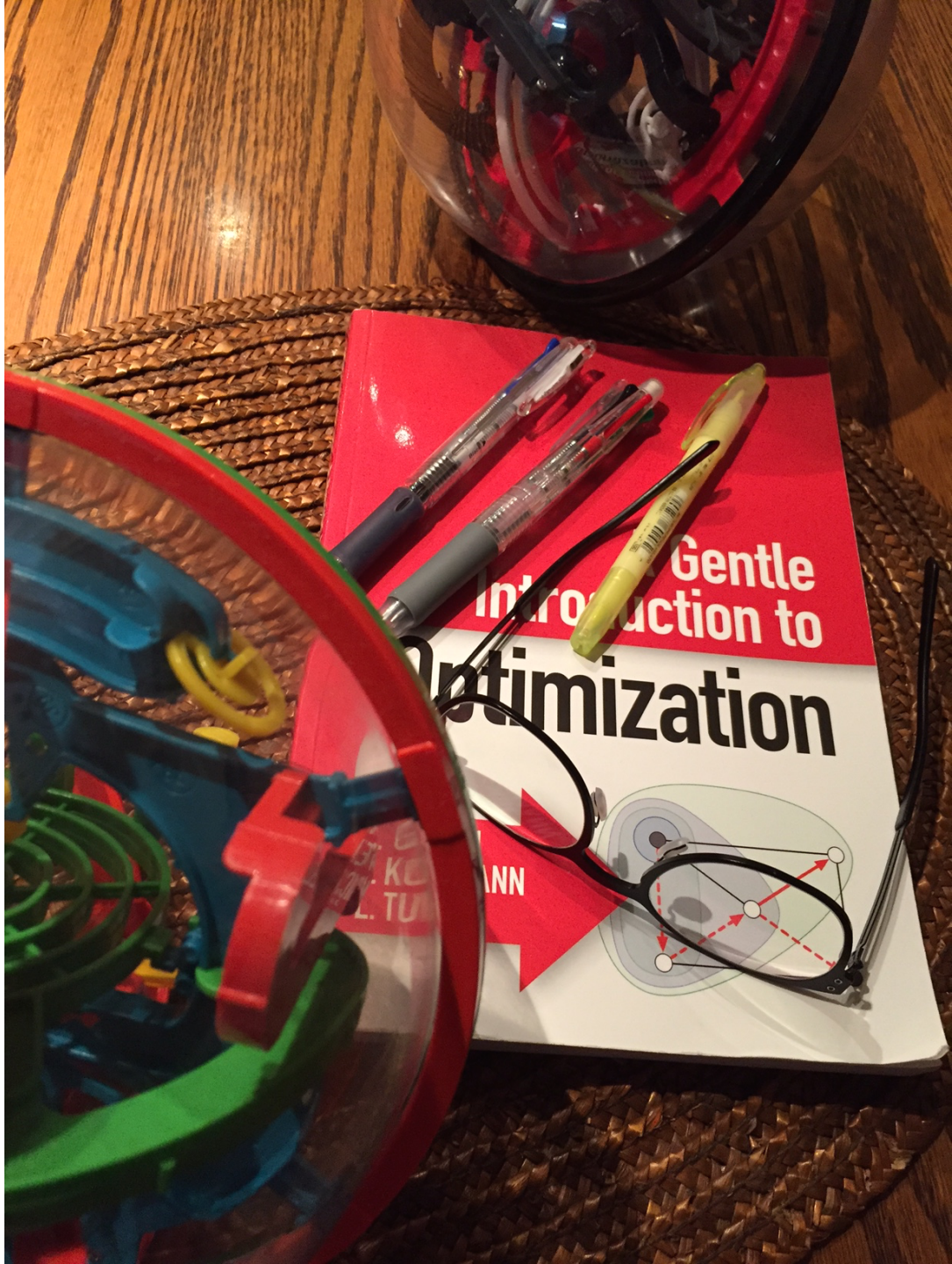
Optimization Theory

Assoc. Prof. Rodney Van Meter

Lecture 1



Scott's Trip

City Navigator North America NT 2017.2

0 50 100 150 200 250 mi

N36° 33.578' W73° 58.540'

WV County Courthouses

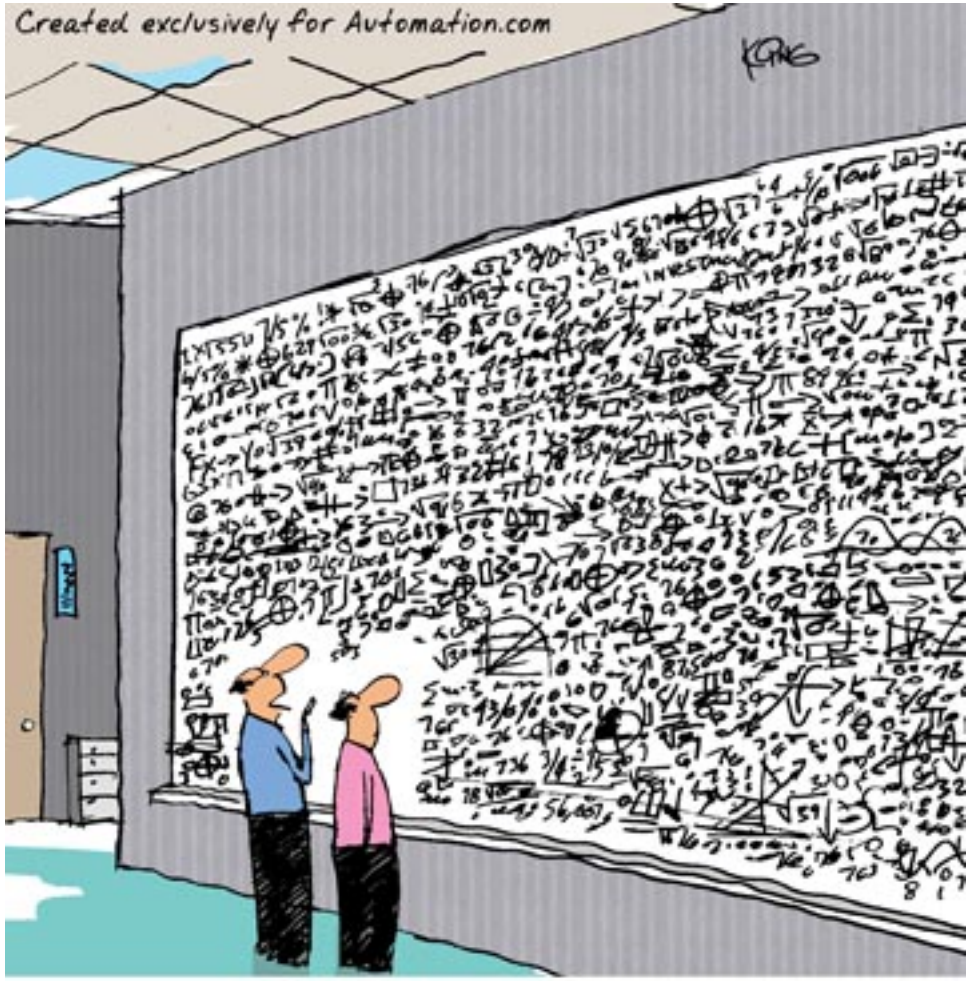
GARMIN

Factory Optimization

http://labs.blogs.com/its_alive_in_the_lab/2010/12/factory-layout-optimization-for-autocad-now-available.html

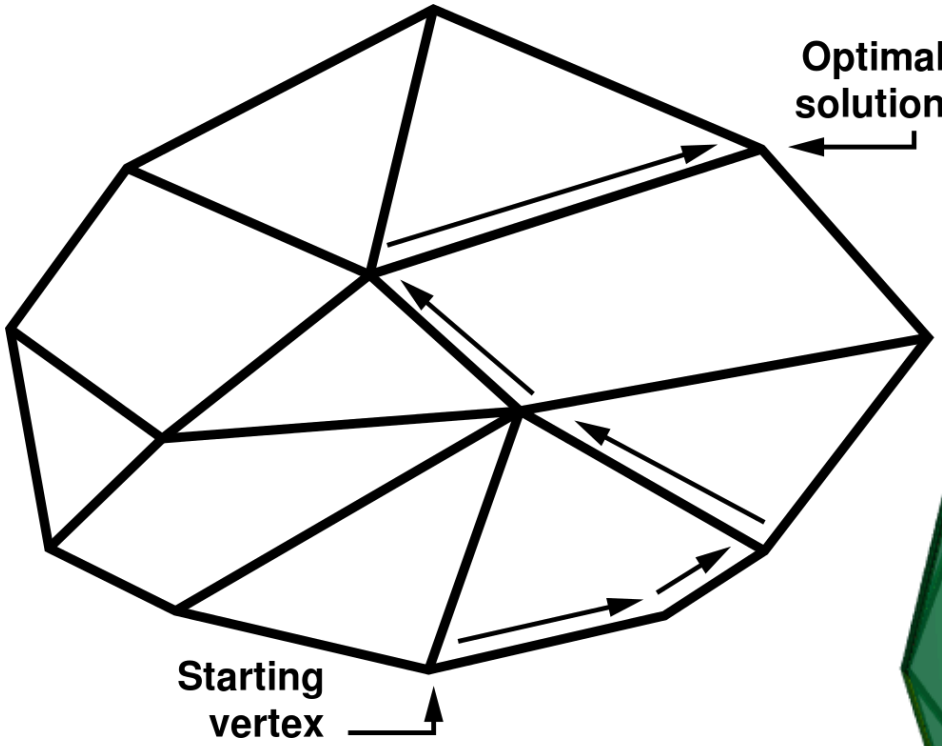
http://www.eesc.st.keio.ac.jp/mech/contents01.html

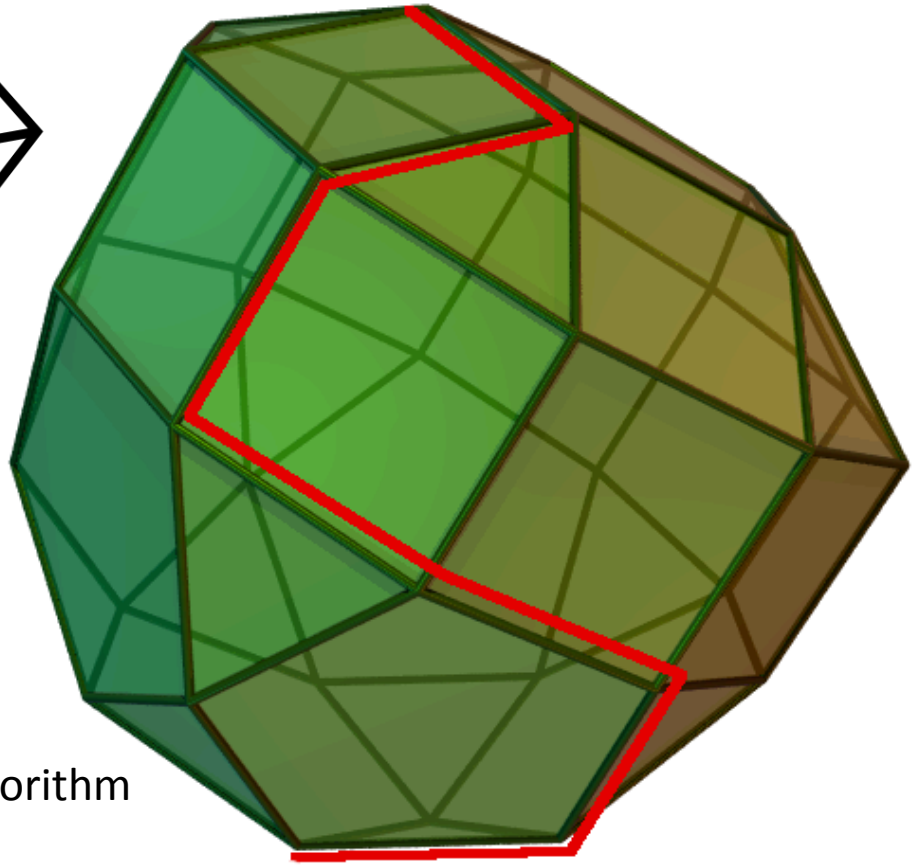
Factory Optimization



"...and that, in simple terms, is my idea on how to increase factory optimization. any questions?"

Simplex Algorithm



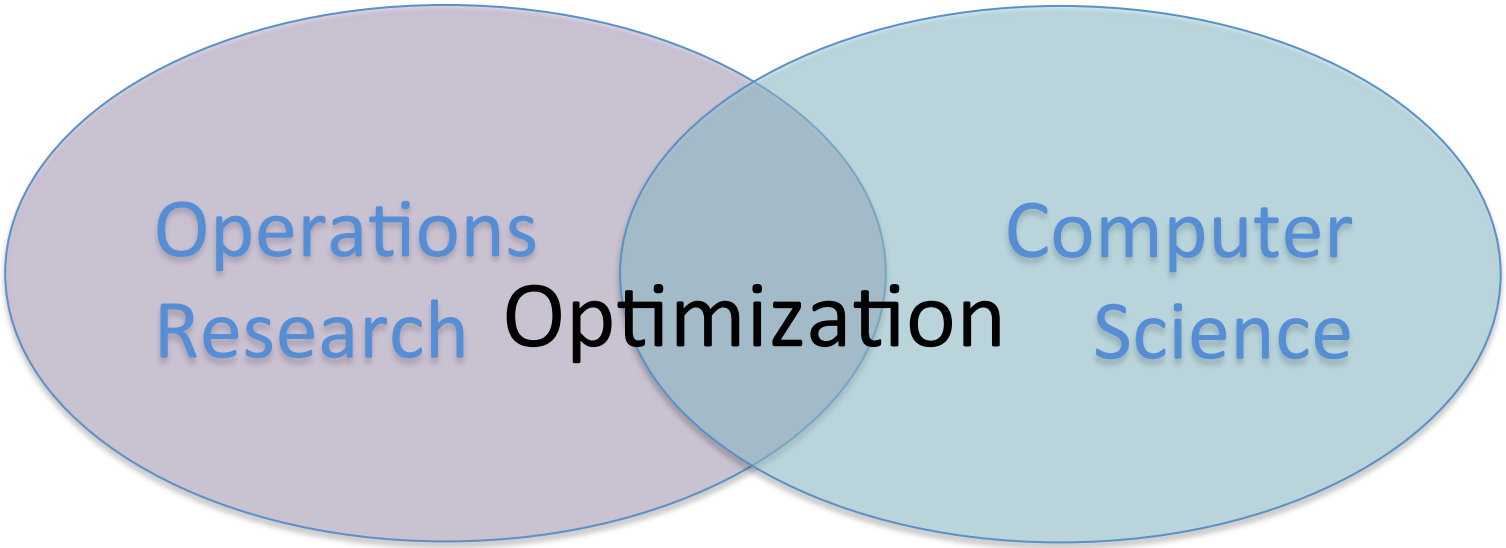


Images from
https://en.wikipedia.org/wiki/Simplex_algorithm

Optimization is Used for...

- Business
- Economics
- Manufacturing
- Networking (Internet)
- Robots (e.g., path planning)
- Biology
- ...pretty much every human activity.

As a Field, It's Related To...



Business
Economics
Manufacturing
Government Policy

Algorithms
Theory
AI
Robotics
Networking

...pretty much every human activity.

What's Hard About Optimization?

- Taking a rough, written description of a problem and turning it into a rigorous set of constraints.
  - Proving that the constraints match the problem!
- Solving the problem itself.
  - Computers are good for this, but only up to a point; where is that point?

Background

Mathematics:

- Vectors & Matrices: addition, multiplication, dot product, basis transformation
- Summation
- Set theory (basics)
- A little geometry
- Functions

Computer science:

- Basic concept of an algorithm: functions, variables, loops, conditions

Types of Programs (Problems)

- Linear Program (LP)
- Integer Program (IP)
- Nonlinear Program (NLP)

Linear Program

function: $f : \mathbb{R}^n \to \mathbb{R}$

affine function: $f(\vec{x}) = \vec{a}^\intercal \vec{x} + \beta$

linear function: $f(\vec{x}) = \vec{a}^\intercal \vec{x}$

linear constraint
(all vars on left, all constants on right):

$$f(\vec{x}) \leq \beta \text{ or } f(\vec{x}) \geq \beta \text{ or } f(\vec{x}) = \beta$$

"A *linear program* (LP) is the problem of maximizing or minimizing an affine function subject to a finite number of linear constraints." GKT, p. 6

What Does All That Mean?

- (Let's do a little linear algebra review...)

Integer Programs

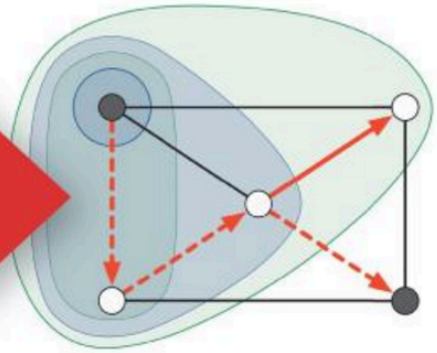
Same thing as LP, except that a non-empty subset of the variables must take integer values

- e.g., if you're packing a shipping container, can't put half a car in one container and half in another!
- In a *pure integer program*, they all have to be integers
- In a *mixed integer program*, only some of them do

Doesn't sound like a big deal, but it is; makes solving the problem *much* harder.

A Gentle Introduction to Optimization

B. GUENIN
J. KÖNEMANN
L. TUNÇEL



Front Cover

This Semester

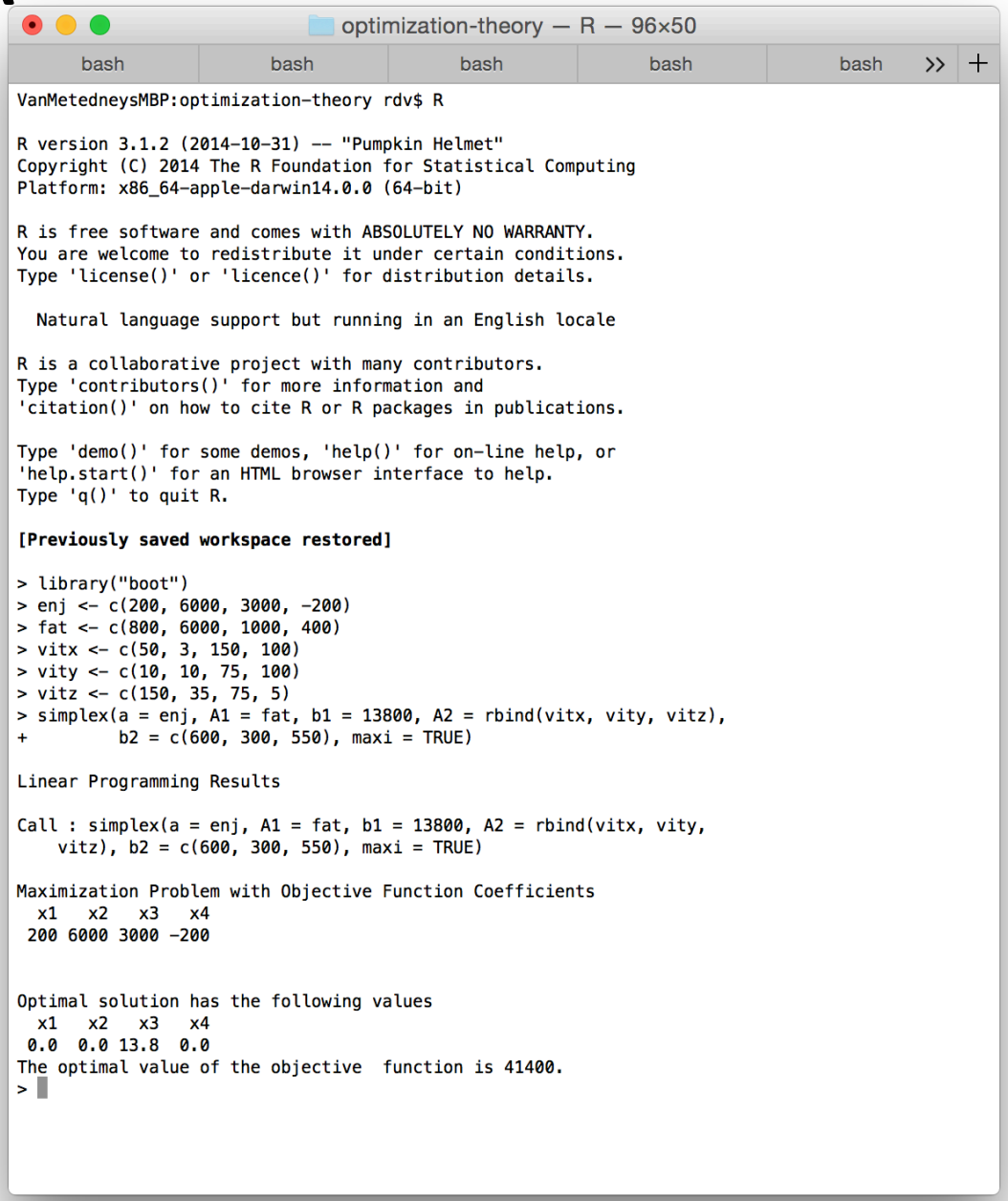
Week/ Chapter	Topic	Week/ Chapter	Topic
1/1	Introduction, Linear/Integer/ Nonlinear Problems, Business/ Manufacturing Examples	8/4.3	Duality Theory 2
2/1.4-1.5	Problems on Graphs and in Networking	9/6.1-6.2	Integer Problems 1
3/2.1-2.4	**Solving Linear Problems: Certificates**	10/6.3-6.4	Integer Problems 2
4/2.5-2.8	**Solving Linear Problems: the Simplex Algorithm**	11/App. A	Computational Complexity
5/3.1	Duality Examples 1: Shortest Path	12/7	Nonlinear Optimization
6/3.2	Duality Examples 2: Minimum Cost Perfect Matching	13/none	Problems in AI and Machine Learning
7/4.1-4.2	Duality Theory 1	14/none	Final Presentations

n.b.: I will be gone on Oct. 18. Class will be rescheduled to Nov. 11.

Programming: R

We will be using R for some problems in this class.

This is a “data science class”, not a “programming class”, but you will be using the computer as a *tool*. Some problems will be too big to do by hand.



Grading

- 40% homeworks
- 40% final project (presentation TBD)
- 20% class participation (means I need to know your name!)

Homeworks

- Target is 1-2 hours/week of regular homework almost every week
- Most will be exercises from the textbook

Final Project

Examples:

- Describe & optimize a significant real-world problem (definition of “significant” TBD)
- Implement simplex algorithm in language of your choice
- Tackle some problems from chapters we didn’t cover (esp. max cut min flow)

Est. 10-20 hours (about half of total homework)