Perfect Matching

December 17, 2016

Abstract

This is pseudocode for Perfect Matching in Bipartite Graphs, adapted from the paperback edition of *A Gentle Introduction to Optimization*, B. Guerin *et al.* This is used as a subroutine to find any perfect matching on a subgraph of the larger bipartite graph in the Hungarian Algorithm, whose goal is to find a minimum weight perfect matching.

The operator Δ is essentially an exclusive OR (XOR) of the two sets:

$$M \Delta P = (M \cup P) \setminus (M \cap P). \tag{1}$$

Executed on an alternating paths of links included in versus excluded from the set and a second path including at leaast part of that first set, it has the behavior of flipping the ones included versus excluded.

I believe this algorithm as stated assumes that the input graph H is connected.

Algorithm 3.4 Perfect Matching

Input : Bipartite graph $H = (V, E)$ with bipartition U,W where $|U| = |W| \geq 1$

Output: A perfect matching M, or a deficient set $B \subseteq U$.

$M := \varnothing$
$T := (\{r\}, \varnothing)$ where $r \in U$ is any M-exposed vertex
while *(1)* **do**
$\{$
    if $\exists$ uv *where* $u \in B(T)$ *and* $v \notin V(T)$ **then**
    $\{$
        if v *is* M*-exposed* **then**
        $\{$
            $P := T_{ru} \cup \{uv\}$
            $M := M \Delta P$
            if M *is a perfect matching* **then** $\{$ **stop**; $\}$
            $T := (\{r\}, \varnothing)$ where $r \in \cup$ is any M-exposed vertex.
        $\}$
        else
        $\{$
            Let $w \in V$ where $vw \in M$
            $T := (V(T) \cup \{v, w\}, E(T) \cup \{uv, vw\})$
        $\}$
    $\}$
    else
    $\{$
        stop $B(T) \subseteq U$ is a deficient set
    $\}$
$\}$