FUNDAMENTALS OF LOGIC
NO.1 WHAT IS LOGIC

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lecture URL

https://vu5.sfc.keio.ac.jp/slide/

Course Summary

- What is the correct deduction?
  - Since A, therefore B.
  - It is either A or B, and it is not A, therefore it should be B.

- Symbolic Logic (Formal Logic, Mathematical Logic)
  - Natural language is difficult to express precise things.
  - Symbols are easy to express things clearly.
  - Logical structure is important.

- Correctness vs Provable
  - We can prove something is correct.
  - Can we prove anything which is correct?

- Various Logical Framework
  - Classical Logic
    - Propositional Logic
    - Predicate Logic
  - Modern Logic
    - Intuitionistic Logic
    - Modal Logic

Logic

Symbolic Logic

Classical Logic

Modern Logic

Propositional Logic

Intuitionistic

Predicate Logic

Modal Logic
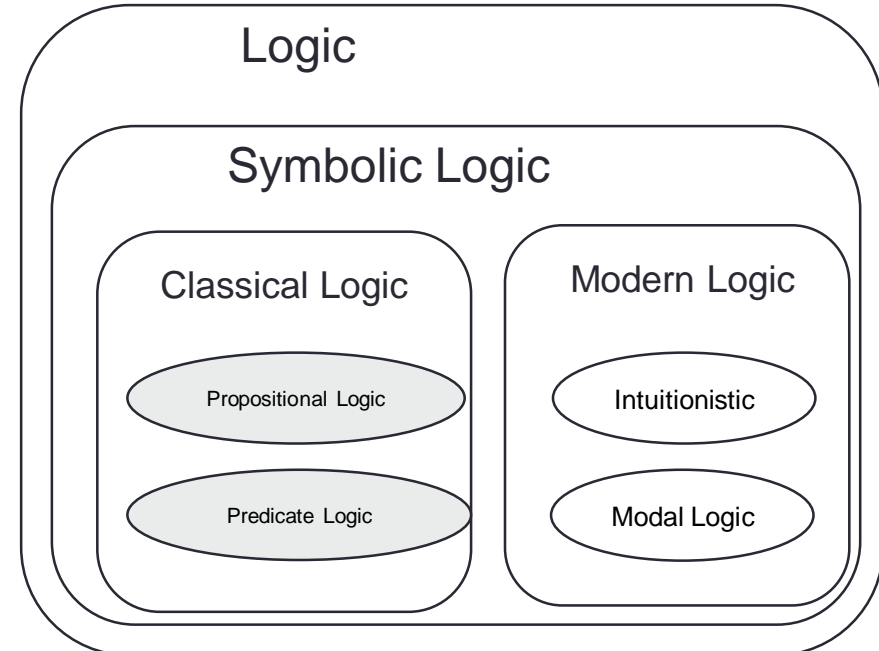
Syllabus

1. What is logic?

2. Proposition and Truth Value
3. Normal Form
4. Proof
5. Sound and Complete
6. Other frameworks

7. Predicate Logic
8. Formula in Predicate Logic
9. Proof in Predicate Logic
10. Herbland Theorem
11. Resolution Principle

12. Modal and Temporal Logic
13. Peano Aruthmetic
14. Summary

What is Logic?

- Logical talking, logical thinking, logical …
  - Not influenced by emotion.
  - Do not assume things which you do not know.
  - Not ambiguous.
  - Based on facts.
  - No jump in the inference.
  - Make the correct thing clear.
  - No hole in the argument.
  - No contradiction.

- Logic
  - Studied in philosophy
  - Studied in mathematics

Logic and Mathematics

Algebra

Linear Algebra

Number Theory

Algebraic Equation

Group

Ring   Field

Geometry

Euclidean Geometry

Non-Euclidean Geometry

Topology

Analysis

Differential Calculus

Integral

Continuous

Differential Equation

Statistics

Probability

Set Theory

Logic

Proof

- Euclid's Elements
  - 13 books about mathematics and geometry.
  - Egypt 300BC

- Axioms of Euclidian Geometry
  1. To draw a straight line from any point to any point.
  2. To produce [extend] a finite straight line continuously in a straight line.
  3. To describe a circle with any centre and distance [radius].
  4. That all right angles are equal to one another.
  5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

- Prove:
  - Alternate interior angles of parallel lines are equal.
  - Sum of interior angles of any triangle is 180 degree.
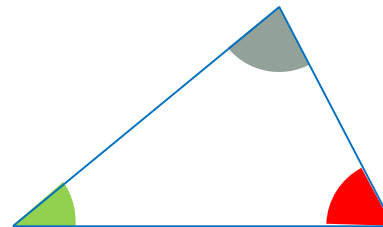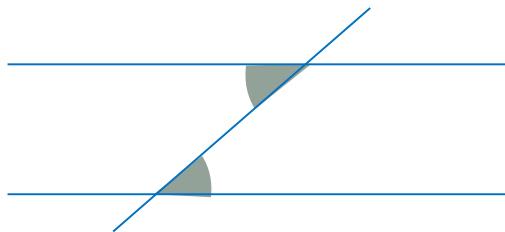
Can you prove?
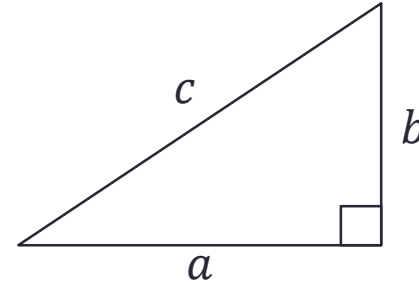
- Pythagorean theorem
  - $a^2 + b^2 = c^2$

- The solution of a quadratic equation $ax^2 + bx + c = 0$ is
  - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- $\sqrt{2}$ is an irrational number.

- π is an irrational number.

- Fundamental theorem of algebra
  - Every non-zero, single-variable, degree n polynomial with complex coefficients has, counted with multiplicity, exactly n complex roots.

- Mean value theorem
  - If a continuous function, f, with an interval, $[a, b]$, as its domain, takes values $f(a)$ and $f(b)$ at each end of the interval, then it also takes any value between $f(a)$ and $f(b)$ at some point within the interval.

Famous Saying in Japan

- If the wind blows, the bucket makers prosper.

  ⬇

  1. If the wind blows, dusts increase.

  ⬇

  2. When dusts hurt eyes, blind people may increase.

  ⬇

  3. If blind people increase, they buy Shamisen.

  ⬇

  4. Since Shamisen needs cat skin, the number of cats decreases.

  ⬇

  5. If the number of cats decreases, the number of rats increases.

  ⬇

  6. If the number of rats increases, they bite buckets.

  ⬇

  7. If buckets are sold more, the bucket makers prosper.

Logic Puzzle

- Logic is often used in puzzles.

  - One of the Cretans said, 'Cretans are always liars'.

  - The prediction which always come true.
    - ``This prediction will not come true.''

  - The problem which cannot be solved.
    - ``This problem cannot be solved.''

  - Barber's paradox
    - There is only one barber in a town.
    - The barber shaves those who do not shave by themselves.
    - The barber do not shave those shave by themselves.
    - Who shaves the barber?

Paradox in Set Theory

- A set is a collection of defined things.
  - Example
    - H: the set of human beings.
    - N: the set of natural numbers.
    - P: the set of statements you thought today.
    - D: the set of things which are not dogs.

- A set may include itself as its member may not include itself as its member.
  - not include itself: H, N
  - include itself: P, D

- Russel's Paradox
  - Let R be the set of all sets that are not members of themselves.
  - Is R a set?

Summary

- Logic
  - Handle deduction or inference

- Mathematical Logic
  - Use symbols

- Logic Framework
  - Propositional logic
  - Predicate logic