FUNDAMENTALS OF LOGIC
NO.1 WHAT IS LOGIC

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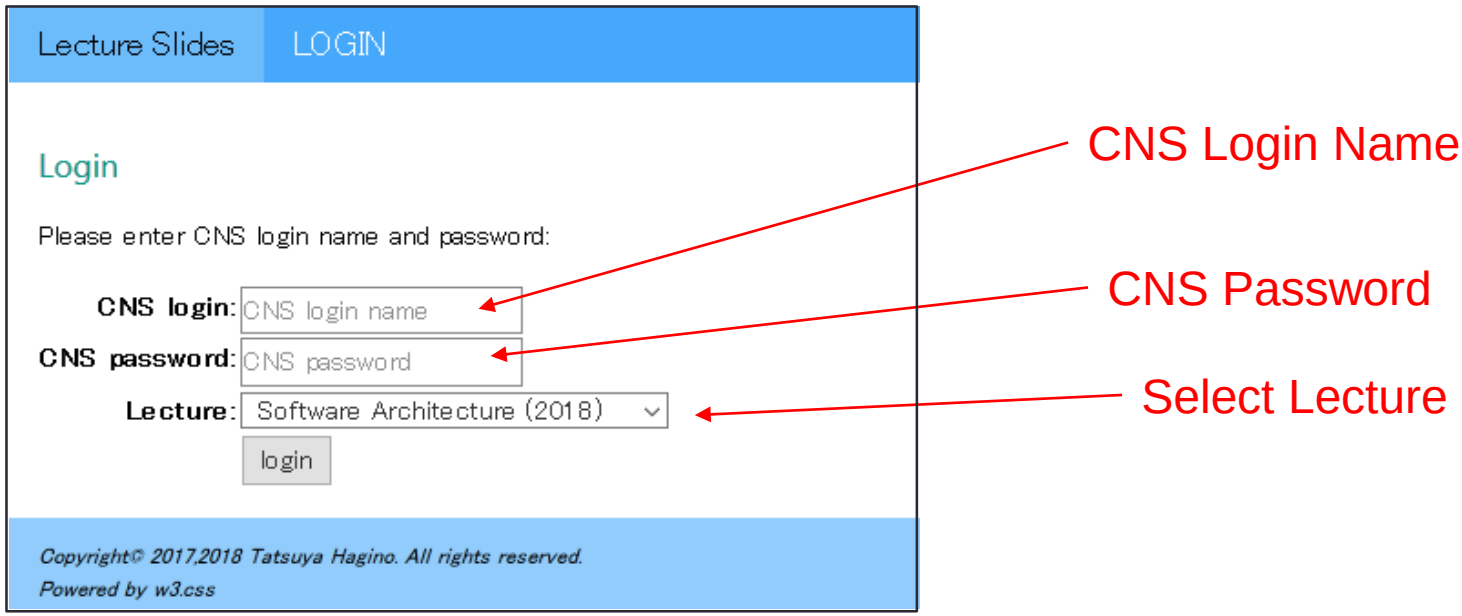
- Select: Fundamentals of Logic



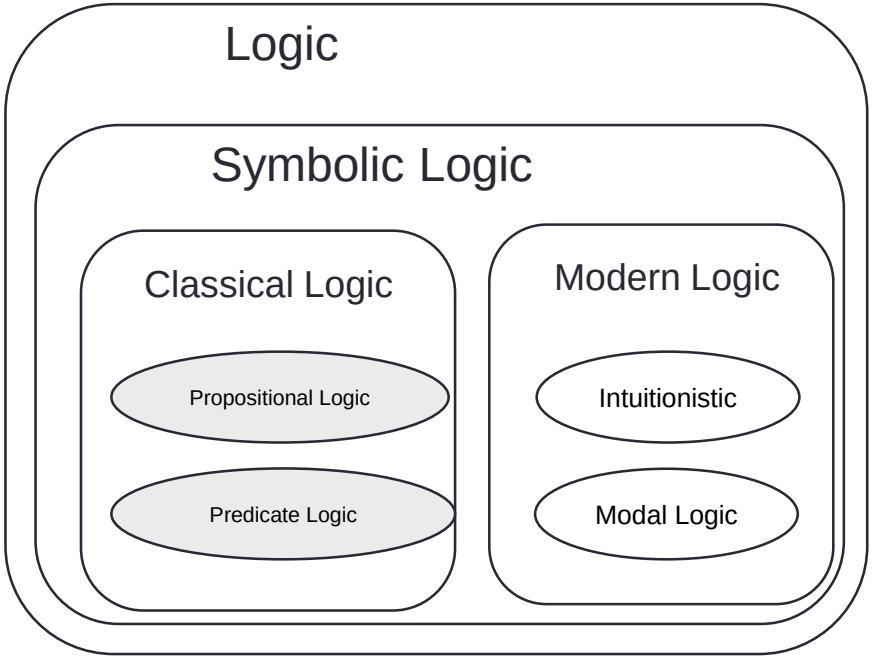
Course Summary

- What is the correct deduction?
  - Since A, therefore B.
  - It is either A or B, and it is not A, therefore it should be B.

- Symbolic Logic (Formal Logic, Mathematical Logic)
  - Natural language is difficult to express precise things.
  - Symbols are easy to express things clearly.
  - Logical structure is important.

- Correctness vs Provable
  - We can prove something is correct.
  - Can we prove anything which is correct?

- Various Logical Framework
  - Classical Logic
    - Propositional Logic
    - Predicate Logic
  - Modern Logic
    - Intuitionistic Logic
    - Modal Logic



Syllabus

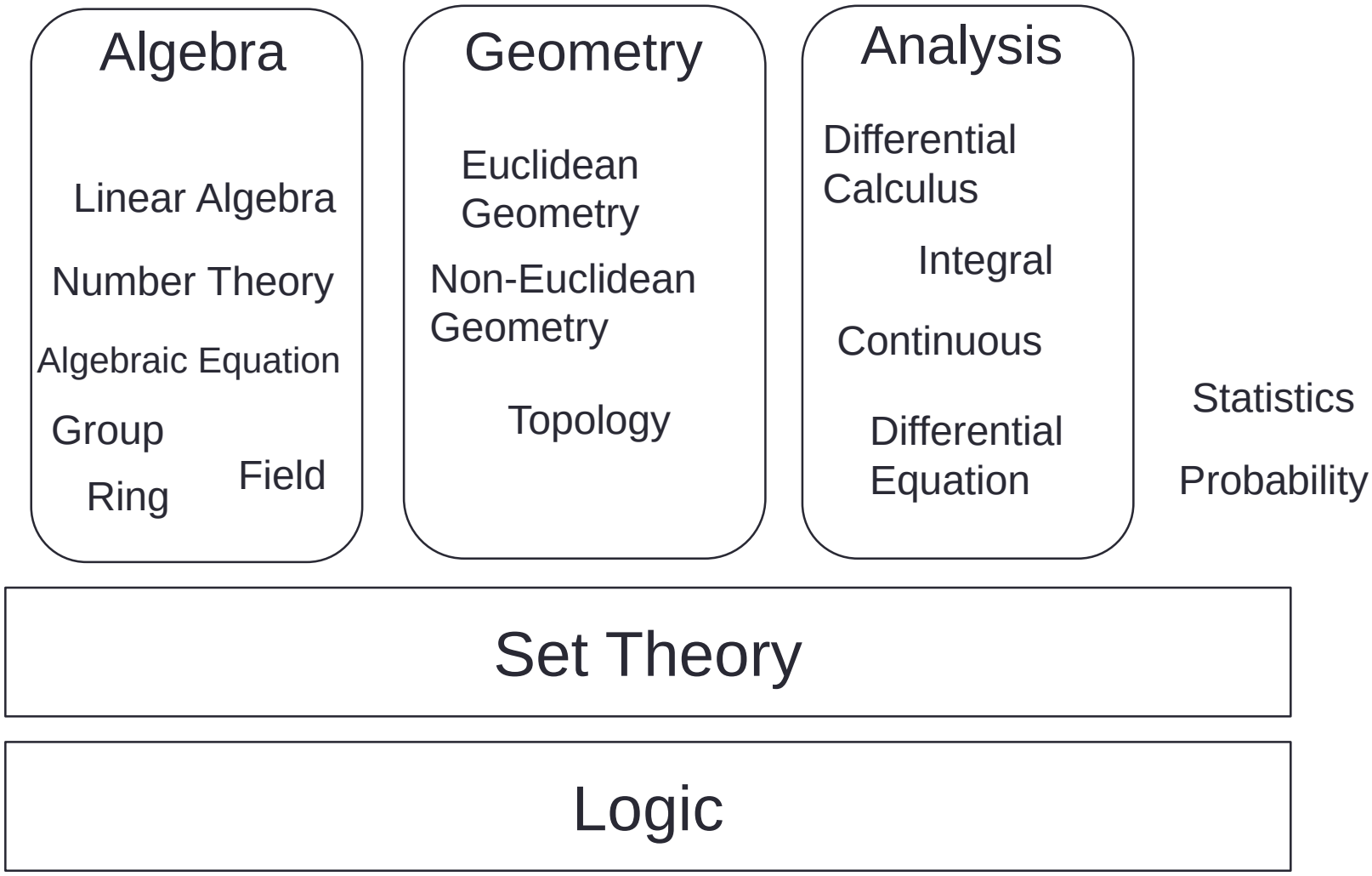
1. What is logic?
2. Proposition and Truth Value
3. Normal Form
4. Proof
5. Proof (exercise)
6. Sound and Complete
7. Other Frameworks
8. Predicate Logic
9. Semantics of Predicate Logic
10. Proof in Predicate Logic
11. Herbland's Theorem
12. Resolution Principle
13. Incompleteness Theorem
14. Other Frameworks

What is Logic?

- Logical talking, logical thinking, logical …
  - Not influenced by emotion.
  - Do not assume things which you do not know.
  - Not ambiguous.
  - Based on facts.
  - No jump in the inference.
  - Make the correct thing clear.
  - No hole in the argument.
  - No contradiction.

- Logic
  - Studied in philosophy
  - Studied in mathematics

Logic and Mathematics

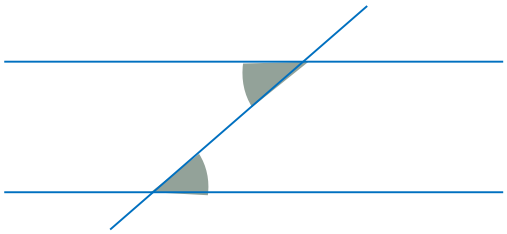


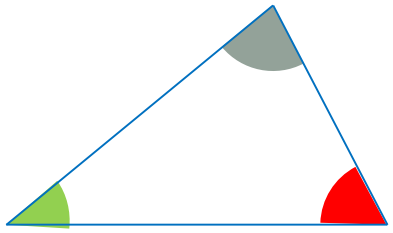
Proof

- Euclid's Elements
  - 13 books about mathematics and geometry.
  - Egypt 300BC

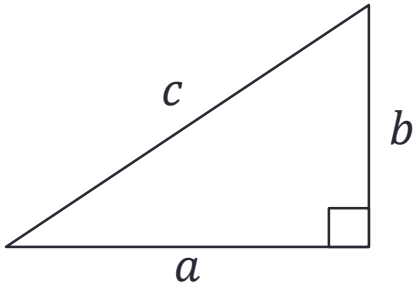
- Axioms of Euclidian Geometry
  1. To draw a straight line from any point to any point.
  2. To produce [extend] a finite straight line continuously in a straight line.
  3. To describe a circle with any center and distance [radius].
  4. That all right angles are equal to one another.
  5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

- Prove:
  - Alternate interior angles of parallel lines are equal.
  - Sum of interior angles of any triangle is 180 degree.





Can you prove?



- Pythagorean theorem
  - $a^2 + b^2 = c^2$
- The solution of a quadratic equation $ax^2 + bx + c = 0$ is
  - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $\sqrt{2}$ is an irrational number.
- π is an irrational number.
- Fundamental theorem of algebra
  - Every non-zero, single-variable, degree n polynomial with complex coefficients has, counted with multiplicity, exactly n complex roots.
- Mean value theorem
  - If a continuous function, f, with an interval, $[a, b]$, as its domain, takes values $f(a)$ and $f(b)$ at each end of the interval, then it also takes any value between $f(a)$ and $f(b)$ at some point within the interval.

Famous Saying in Japan

- If the wind blows, the bucket makers prosper.

1. If the wind blows, dusts increase.
2. When dusts hurt eyes, blind people may increase.
3. If blind people increase, they buy Shamisen.
4. Since Shamisen needs cat skin, the number of cats decreases.
5. If the number of cats decreases, the number of rats increases.
6. If the number of rats increases, they bite buckets.
7. If buckets are sold more, the bucket makers prosper.

Logic Puzzle

- Logic is often used in puzzles.
  - One of the Cretans said, 'Cretans are always liars'.
  - The prediction which always come true.
    - ``This prediction will not come true.''
  - The problem which cannot be solved.
    - ``This problem cannot be solved.''
  - Barber's paradox
    - There is only one barber in a town.
    - The barber shaves those who do not shave by themselves.
    - The barber do not shave those shave by themselves.
    - Who shaves the barber?

Paradox in Set Theory

- A set is a collection of defined things.
  - Example
    - H: the set of human beings.
    - N: the set of natural numbers.
    - P: the set of statements you thought today.
    - D: the set of things which are not dogs.
- A set may include itself as its member may not include itself as its member.
  - not include itself: H, N
  - include itself: P, D
- Russel's Paradox
  - Let R be the set of all sets that are not members of themselves.
  - Is R a set?

Summary

- Logic
  - Handle deduction or inference
- Mathematical Logic
  - Use symbols
- Logic Framework
  - Propositional logic
  - Predicate logic