LK Axioms and Inference Rules

- Axioms:

$$\frac{}{A \vdash A} \; (I) \qquad\qquad \frac{}{\vdash \top} \; (\top) \qquad\qquad \frac{}{\bot \vdash} \; (\bot)$$

- Inference Rules for Structure:

$$\frac{\Gamma \vdash \Delta}{A, \Gamma \vdash \Delta} \; (WL) \qquad\qquad\qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A} \; (WR)$$

$$\frac{A, A, \Gamma \vdash \Delta}{A, \Gamma \vdash \Delta} \; (CL) \qquad\qquad\qquad \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A} \; (CR)$$

$$\frac{\Gamma_1, A, B, \Gamma_2 \vdash \Delta}{\Gamma_1, B, A, \Gamma_2 \vdash \Delta} \; (EL) \qquad\qquad \frac{\Gamma \vdash \Delta_1, A, B, \Delta_2}{\Gamma \vdash \Delta_1, B, A, \Delta_2} \; (ER)$$

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad A, \Gamma_2 \vdash \Delta_2}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \; (Cut)$$

(where Γ, Δ are sequence of logical formulae)

- Inference Rules for Logical Connectives:

$$\frac{A, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \; (\wedge L_1) \qquad\qquad \frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B} \; (\vee R_1)$$

$$\frac{B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \; (\wedge L_2) \qquad\qquad \frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \vee B} \; (\vee R_2)$$

$$\frac{A, \Gamma_1 \vdash \Delta_1 \quad B, \Gamma_2 \vdash \Delta_2}{A \vee B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \; (\vee L) \qquad \frac{\Gamma_1 \vdash \Delta_1, A \quad \Gamma_2 \vdash \Delta_2, B}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2, A \wedge B} \; (\wedge R)$$

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad B, \Gamma_2 \vdash \Delta_2}{A \rightarrow B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \; (\rightarrow L) \qquad \frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \rightarrow B} \; (\rightarrow R)$$

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta} \; (\neg L) \qquad\qquad \frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A} \; (\neg R)$$

(where Γ, Δ are sequence of logical formulae)