

# LKの公理と推論規則

- 公理

$$\frac{}{A \vdash A} \text{ (I)} \quad \frac{}{\vdash \top} \text{ (T)} \quad \frac{}{\perp \vdash} \text{ (⊥)}$$

- 構造に関する推論規則

$$\frac{\Gamma \vdash \Delta}{A, \Gamma \vdash \Delta} \text{ (WL)}$$

$$\frac{A, A, \Gamma \vdash \Delta}{A, \Gamma \vdash \Delta} \text{ (CL)}$$

$$\frac{\Gamma_1, A, B, \Gamma_2 \vdash \Delta}{\Gamma_1, B, A, \Gamma_2 \vdash \Delta} \text{ (EL)}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A} \text{ (WR)}$$

$$\frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A} \text{ (CR)}$$

$$\frac{\Gamma \vdash \Delta_1, A, B, \Delta_2}{\Gamma \vdash \Delta_1, B, A, \Delta_2} \text{ (ER)}$$

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad A, \Gamma_2 \vdash \Delta_2}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \text{ (Cut)}$$

(ここで $\Gamma, \Delta$ は論理式の列)

- 論理結合子に関する推論規則

$$\frac{A, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \text{ (\wedge L<sub>1</sub>)}$$

$$\frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B} \text{ (\vee R<sub>1</sub>)}$$

$$\frac{B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \text{ (\wedge L<sub>2</sub>)}$$

$$\frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \vee B} \text{ (\vee R<sub>2</sub>)}$$

$$\frac{A, \Gamma_1 \vdash \Delta_1 \quad B, \Gamma_2 \vdash \Delta_2}{A \vee B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \text{ (\vee L)}$$

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad \Gamma_2 \vdash \Delta_2, B}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2, A \wedge B} \text{ (\wedge R)}$$

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad B, \Gamma_2 \vdash \Delta_2}{A \rightarrow B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \text{ (\rightarrow L)}$$

$$\frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \rightarrow B} \text{ (\rightarrow R)}$$

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta} \text{ (\neg L)}$$

$$\frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A} \text{ (\neg R)}$$

(ここで $\Gamma, \Delta$ は論理式の列)