論理学
第6回補足「分解と証明」

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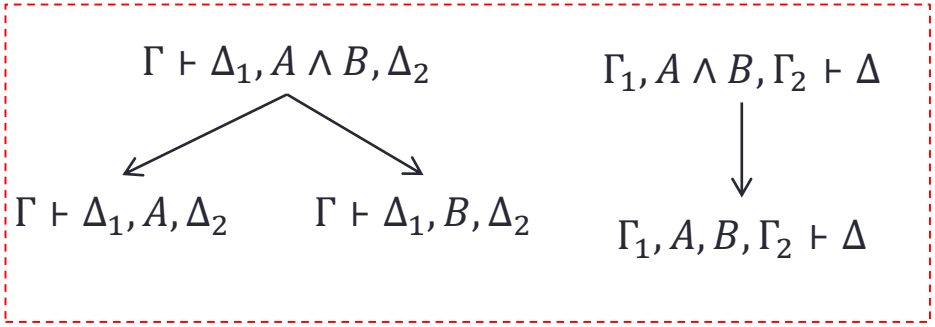
lecture URL

https://vu5.sfc.keio.ac.jp/slide/

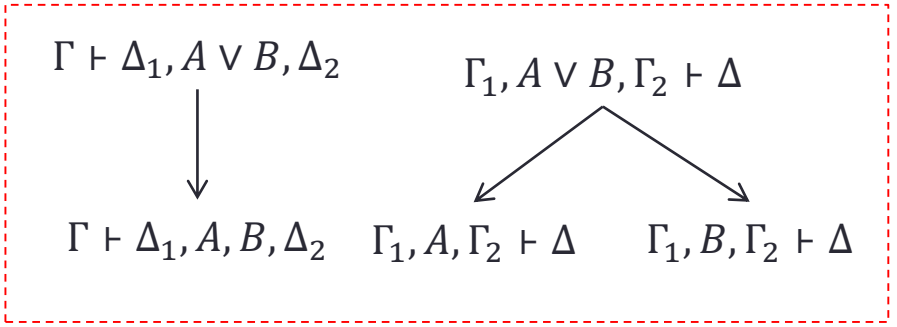
式の分解

- 式の分解を次のように定義する.

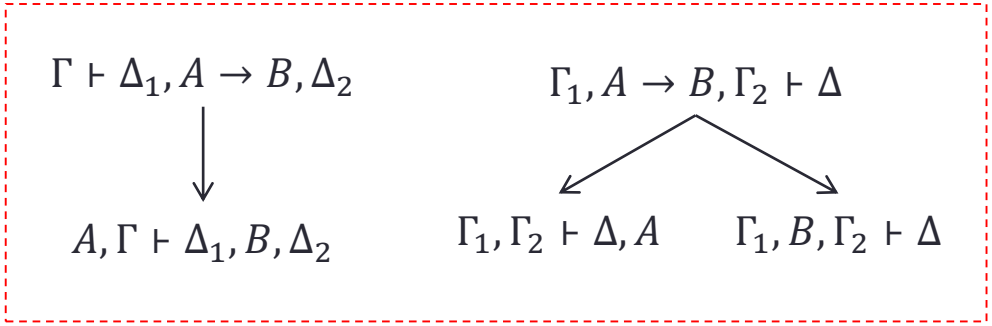
かつ



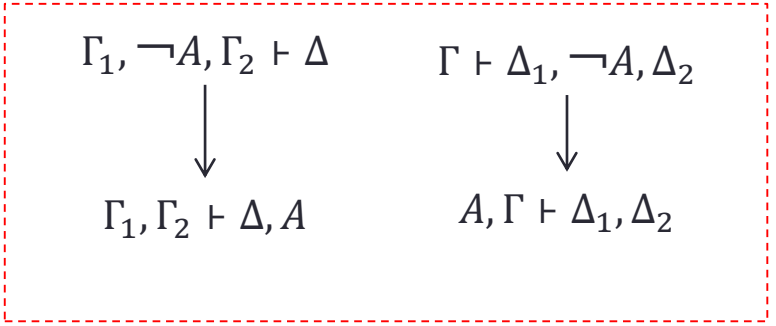
または



ならば

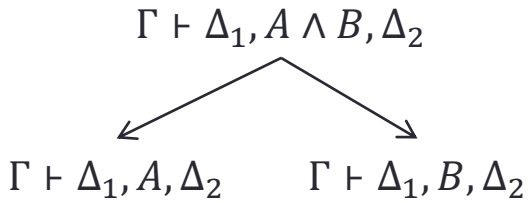


否定



分解と証明の関係（かつ，後件）

分解



$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \vdash \Delta_1, A, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, A}(\mathrm{ER}) \quad \dfrac{\Gamma \vdash \Delta_1, B, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, B}(\mathrm{ER})}{\Gamma, \Gamma \vdash \Delta_1, \Delta_2, \Delta_1, \Delta_2, A \wedge B}(\wedge\mathrm{R})}{\Gamma \vdash \Delta_1, \Delta_2, \Delta_1, \Delta_2, A \wedge B}(\mathrm{CL})}{\Gamma \vdash A \wedge B, \Delta_1, \Delta_1, \Delta_2, \Delta_2}(\mathrm{ER})}{\Gamma \vdash A \wedge B, \Delta_1, \Delta_1, \Delta_2}(\mathrm{CR})}{\Gamma \vdash A \wedge B, \Delta_2, \Delta_1, \Delta_1}(\mathrm{ER})}{\Gamma \vdash A \wedge B, \Delta_2, \Delta_1}(\mathrm{CR})}{\Gamma \vdash \Delta_1, A \wedge B, \Delta_2}(\mathrm{ER})$$

推論規則

$$\dfrac{\Gamma_1 \vdash \Delta_1, A \quad \Gamma_2 \vdash \Delta_2, B}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2, A \wedge B}(\wedge\mathrm{R})$$

分解と証明の関係（かつ，前件）

分解

$$\Gamma_1, A \wedge B, \Gamma_2 \vdash \Delta$$

$$\downarrow$$

$$\Gamma_1, A, B, \Gamma_2 \vdash \Delta$$

$\Leftrightarrow$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma_1, A, B, \Gamma_2 \vdash \Delta}{A, B, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\text{EL})}{A \wedge B, B, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\wedge L_1)}{B, A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\text{EL})}{A \wedge B, A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\wedge L_2)}{A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\text{CL})$$

$$\dfrac{}{\Gamma_1, A \wedge B, \Gamma_2 \vdash \Delta}\,(\text{EL})$$

推論規則

$$\dfrac{A, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta}\,(\wedge L_1) \qquad \dfrac{B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta}\,(\wedge L_2)$$

分解と証明の関係（または，後件）

分解

$$\Gamma \vdash \Delta_1, A \vee B, \Delta_2 \quad \downarrow \quad \Gamma \vdash \Delta_1, A, B, \Delta_2$$

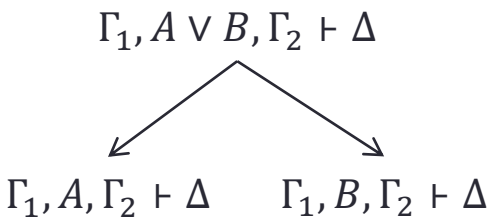
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \vdash \Delta_1, A, B, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, A, B}\,(\text{ER})}{\Gamma \vdash \Delta_1, \Delta_2, A, A \vee B}\,(\vee R_2)}{\Gamma \vdash \Delta_1, \Delta_2, A \vee B, A}\,(\text{ER})}{\Gamma \vdash \Delta_1, \Delta_2, A \vee B, A \vee B}\,(\vee R_1)}{\Gamma \vdash \Delta_1, \Delta_2, A \vee B}\,(\text{CR})}{\Gamma \vdash \Delta_1, A \vee B, \Delta_2}\,(\text{ER})$$

推論規則

$$\dfrac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B}\,(\vee R_1) \qquad \dfrac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \vee B}\,(\vee R_2)$$

分解と証明の関係（または，前件）

分解



$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma_1, A, \Gamma_2 \vdash \Delta}{A, \Gamma_1, \Gamma_2 \vdash \Delta}(\mathrm{EL}) \quad \dfrac{\Gamma_1, B, \Gamma_2 \vdash \Delta}{B, \Gamma_1, \Gamma_2 \vdash \Delta}(\mathrm{EL})}{A \vee B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta, \Delta}(\vee\,\mathrm{L})}{A \vee B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta}(\mathrm{CR})}{\Gamma_1, \Gamma_1, \Gamma_2, \Gamma_2, A \vee B \vdash \Delta}(\mathrm{EL})}{\Gamma_1, \Gamma_2, \Gamma_2, A \vee B \vdash \Delta}(\mathrm{CL})}{\Gamma_2, \Gamma_2, \Gamma_1, A \vee B \vdash \Delta}(\mathrm{EL})}{\Gamma_2, \Gamma_1, A \vee B \vdash \Delta}(\mathrm{CL})}{\Gamma_1, A \vee B, \Gamma_2 \vdash \Delta}(\mathrm{EL})$$

推論規則

$$\dfrac{A, \Gamma_1 \vdash \Delta_1 \qquad B, \Gamma_2 \vdash \Delta_2}{A \vee B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2}(\vee\mathrm{L})$$

分解と証明の関係（ならば，後件）

分解

$$\Gamma \vdash \Delta_1, A \to B, \Delta_2$$

$$\downarrow$$

$$A, \Gamma \vdash \Delta_1, B, \Delta_2$$

$$\dfrac{\dfrac{\dfrac{A, \Gamma \vdash \Delta_1, B, \Delta_2}{A, \Gamma \vdash \Delta_1, \Delta_2, B}\,(\text{ER})}{\Gamma \vdash \Delta_1, \Delta_2, A \to B}\,(\to\text{R})}{\Gamma \vdash \Delta_1, A \to B, \Delta_2}\,(\text{ER})$$

推論規則

$$\dfrac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \to B}\,(\to\text{R})$$

分解と証明の関係（ならば，前件）

分解

$$\Gamma_1, A \to B, \Gamma_2 \vdash \Delta$$

$$\Gamma_1, \Gamma_2 \vdash \Delta, A \qquad \Gamma_1, B, \Gamma_2 \vdash \Delta$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma_1, \Gamma_2 \vdash \Delta, A \qquad \dfrac{\Gamma_1, B, \Gamma_2 \vdash \Delta}{B, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\mathrm{EL})}{A \to B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta, \Delta}\,(\to \mathrm{L})}{A \to B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta}\,(\mathrm{CR})}{\Gamma_1, \Gamma_1, \Gamma_2, \Gamma_2, A \to B \vdash \Delta}\,(\mathrm{EL})}{\Gamma_1, \Gamma_2, \Gamma_2, A \to B \vdash \Delta}\,(\mathrm{CL})}{\Gamma_2, \Gamma_2, \Gamma_1, A \to B \vdash \Delta}\,(\mathrm{EL})}{\Gamma_2, \Gamma_1, A \to B \vdash \Delta}\,(\mathrm{CL})}{\Gamma_1, A \to B, \Gamma_2 \vdash \Delta}\,(\mathrm{EL})$$

推論規則

$$\dfrac{\Gamma_1 \vdash \Delta_1, A \qquad B, \Gamma_2 \vdash \Delta_2}{A \to B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2}\,(\to \mathrm{L})$$

分解と証明の関係（否定，後件）

分解

$$\Gamma \vdash \Delta_1, \neg A, \Delta_2 \quad \Downarrow \quad A, \Gamma \vdash \Delta_1, \Delta_2$$

$$\dfrac{\dfrac{A, \Gamma \vdash \Delta_1, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, \neg A}\ (\neg R)}{\Gamma \vdash \Delta_1, \neg A, \Delta_2}\ (ER)$$

推論規則

$$\dfrac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A}\ (\neg R)$$

分解と証明の関係（否定，前件）

分解

$$\Gamma_1, \neg A, \Gamma_2 \vdash \Delta$$
$$\downarrow$$
$$\Gamma_1, \Gamma_2 \vdash \Delta, A$$

$$\dfrac{\dfrac{\Gamma_1, \Gamma_2 \vdash \Delta, A}{\neg A, \Gamma_1, \Gamma_2 \vdash \Delta}\ (\text{EL})}{\Gamma_1, \neg A, \Gamma_2 \vdash \Delta}\ (\neg\text{L})$$

推論規則

$$\dfrac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta}\ (\neg\text{L})$$