

論理学

第6回補足「分解と証明」

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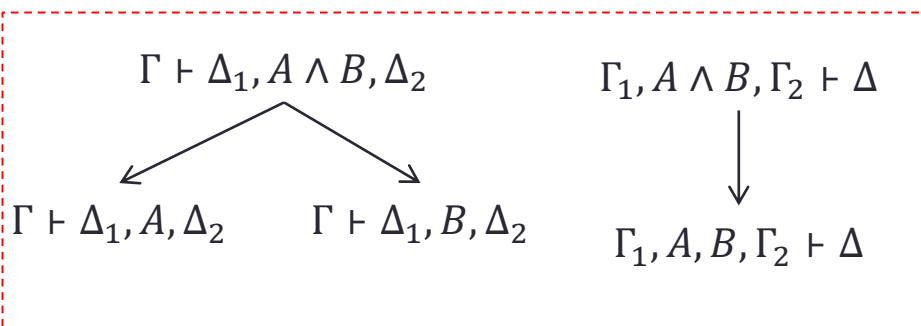
lecture URL

<https://vu5.sfc.keio.ac.jp/slide/>

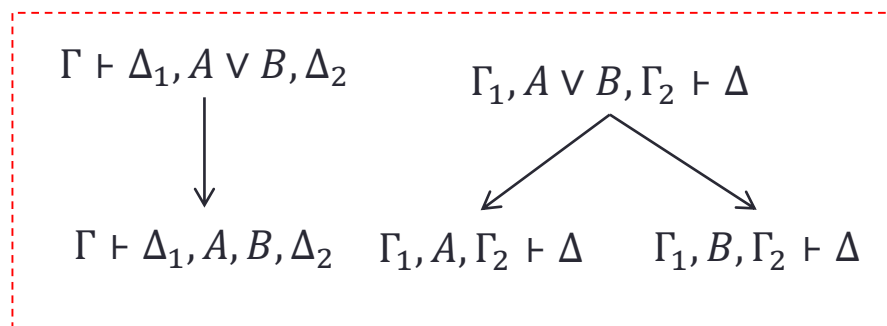
式の分解

- 式の分解を次のように定義する.

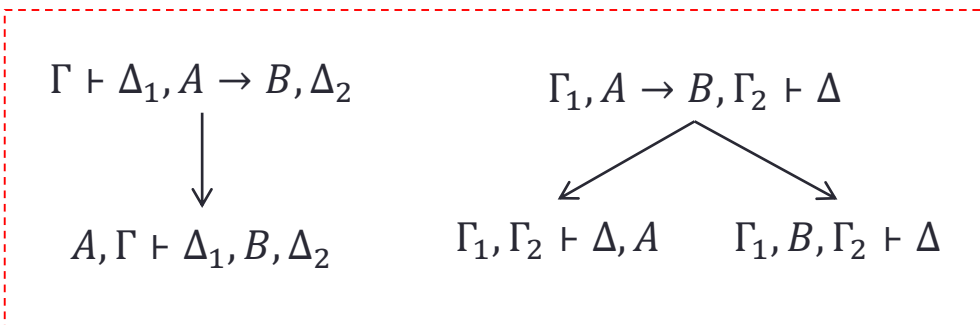
かつ



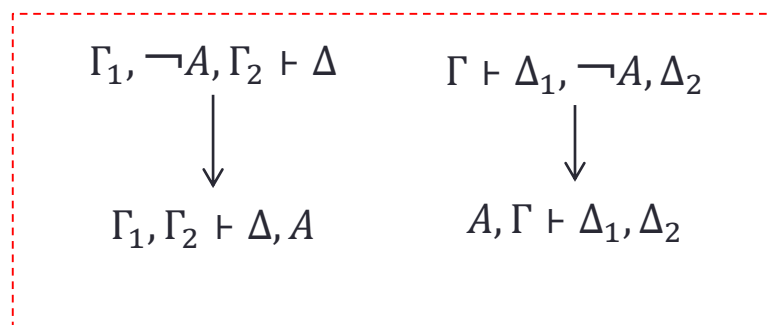
または



ならば

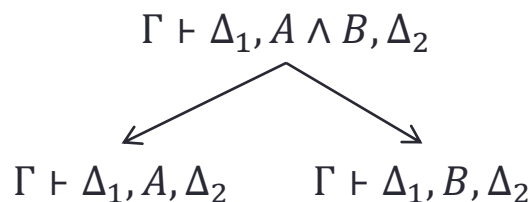


否定



分解と証明の関係(かつ, 後件)

分解



$$\begin{array}{c}
 \frac{\Gamma \vdash \Delta_1, A, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, A} \text{ (ER)} \quad \frac{\Gamma \vdash \Delta_1, B, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, B} \text{ (ER)} \\
 \hline
 \Gamma, \Gamma \vdash \Delta_1, \Delta_2, \Delta_1, \Delta_2, A \wedge B \quad \text{(\wedge R)} \\
 \hline
 \Gamma \vdash \Delta_1, \Delta_2, \Delta_1, \Delta_2, A \wedge B \quad \text{(CL)} \\
 \hline
 \Gamma \vdash A \wedge B, \Delta_1, \Delta_1, \Delta_2, \Delta_2 \quad \text{(ER)} \\
 \hline
 \Gamma \vdash A \wedge B, \Delta_1, \Delta_1, \Delta_2 \quad \text{(CR)} \\
 \hline
 \Gamma \vdash A \wedge B, \Delta_2, \Delta_1, \Delta_1 \quad \text{(ER)} \\
 \hline
 \Gamma \vdash A \wedge B, \Delta_2, \Delta_1 \quad \text{(CR)} \\
 \hline
 \Gamma \vdash \Delta_1, A \wedge B, \Delta_2 \quad \text{(ER)}
 \end{array}$$

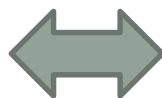
推論規則

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad \Gamma_2 \vdash \Delta_2, B}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2, A \wedge B} \text{ (\wedge R)}$$

分解と証明の関係(かつ, 前件)

分解

$$\begin{array}{c} \Gamma_1, A \wedge B, \Gamma_2 \vdash \Delta \\ \downarrow \\ \Gamma_1, A, B, \Gamma_2 \vdash \Delta \end{array}$$



$$\begin{array}{c} \frac{\Gamma_1, A, B, \Gamma_2 \vdash \Delta}{A, B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (EL)} \\ \frac{A, B, \Gamma_1, \Gamma_2 \vdash \Delta}{A \wedge B, B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (}\wedge L_1\text{)} \\ \frac{A \wedge B, B, \Gamma_1, \Gamma_2 \vdash \Delta}{B, A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (EL)} \\ \frac{B, A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta}{A \wedge B, A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (}\wedge L_2\text{)} \\ \frac{A \wedge B, A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta}{A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (CL)} \\ \frac{A \wedge B, \Gamma_1, \Gamma_2 \vdash \Delta}{\Gamma_1, A \wedge B, \Gamma_2 \vdash \Delta} \text{ (EL)} \end{array}$$

推論規則

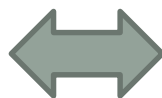
$$\frac{A, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \text{ (}\wedge L_1\text{)}$$

$$\frac{B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \text{ (}\wedge L_2\text{)}$$

分解と証明の関係(または, 後件)

分解

$$\begin{array}{c} \Gamma \vdash \Delta_1, A \vee B, \Delta_2 \\ \downarrow \\ \Gamma \vdash \Delta_1, A, B, \Delta_2 \end{array}$$



$$\begin{array}{c} \frac{\Gamma \vdash \Delta_1, A, B, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, A, B} \text{ (ER)} \\ \frac{\Gamma \vdash \Delta_1, \Delta_2, A, B}{\Gamma \vdash \Delta_1, \Delta_2, A, A \vee B} \text{ (VR}_2\text{)} \\ \frac{\Gamma \vdash \Delta_1, \Delta_2, A, A \vee B}{\Gamma \vdash \Delta_1, \Delta_2, A \vee B, A} \text{ (ER)} \\ \frac{\Gamma \vdash \Delta_1, \Delta_2, A \vee B, A}{\Gamma \vdash \Delta_1, \Delta_2, A \vee B, A \vee B} \text{ (VR}_1\text{)} \\ \frac{\Gamma \vdash \Delta_1, \Delta_2, A \vee B, A \vee B}{\Gamma \vdash \Delta_1, \Delta_2, A \vee B} \text{ (CR)} \\ \frac{\Gamma \vdash \Delta_1, \Delta_2, A \vee B}{\Gamma \vdash \Delta_1, A \vee B, \Delta_2} \text{ (ER)} \end{array}$$

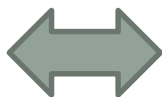
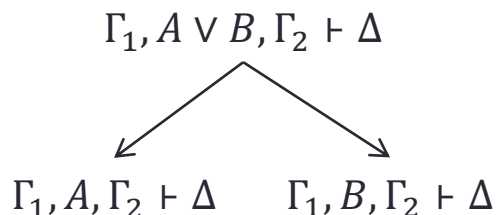
推論規則

$$\frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B} \text{ (VR}_1\text{)}$$

$$\frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \vee B} \text{ (VR}_2\text{)}$$

分解と証明の関係(または, 前件)

分解



$$\begin{array}{c}
 \text{(EL)} \frac{\Gamma_1, A, \Gamma_2 \vdash \Delta}{A, \Gamma_1, \Gamma_2 \vdash \Delta} \quad \frac{\Gamma_1, B, \Gamma_2 \vdash \Delta}{B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{(EL)} \\
 \hline
 \frac{A \vee B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta, \Delta}{A \vee B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta} \text{(CR)} \\
 \hline
 \frac{A \vee B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta}{\Gamma_1, \Gamma_1, \Gamma_2, \Gamma_2, A \vee B \vdash \Delta} \text{(EL)} \\
 \hline
 \frac{\Gamma_1, \Gamma_1, \Gamma_2, \Gamma_2, A \vee B \vdash \Delta}{\Gamma_1, \Gamma_2, \Gamma_2, A \vee B \vdash \Delta} \text{(CL)} \\
 \hline
 \frac{\Gamma_1, \Gamma_2, \Gamma_2, A \vee B \vdash \Delta}{\Gamma_2, \Gamma_2, \Gamma_1, A \vee B \vdash \Delta} \text{(EL)} \\
 \hline
 \frac{\Gamma_2, \Gamma_2, \Gamma_1, A \vee B \vdash \Delta}{\Gamma_2, \Gamma_1, A \vee B \vdash \Delta} \text{(CL)} \\
 \hline
 \frac{\Gamma_2, \Gamma_1, A \vee B \vdash \Delta}{\Gamma_1, A \vee B, \Gamma_2 \vdash \Delta} \text{(EL)}
 \end{array}$$

推論規則

$$\frac{A, \Gamma_1 \vdash \Delta_1 \quad B, \Gamma_2 \vdash \Delta_2}{A \vee B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \text{(VL)}$$

分解と証明の関係(ならば, 後件)

分解

$$\begin{array}{c} \Gamma \vdash \Delta_1, A \rightarrow B, \Delta_2 \\ \downarrow \\ A, \Gamma \vdash \Delta_1, B, \Delta_2 \end{array}$$



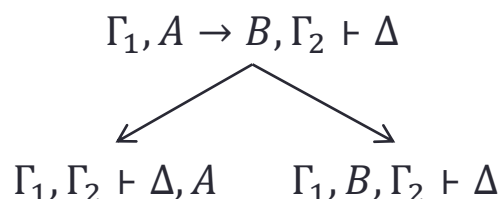
$$\frac{\frac{\frac{A, \Gamma \vdash \Delta_1, B, \Delta_2}{A, \Gamma \vdash \Delta_1, \Delta_2, B} \text{ (ER)}}{\Gamma \vdash \Delta_1, \Delta_2, A \rightarrow B} \text{ (}\rightarrow\text{R)}}{\Gamma \vdash \Delta_1, A \rightarrow B, \Delta_2} \text{ (ER)}$$

推論規則

$$\frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \rightarrow B} \text{ (}\rightarrow\text{R)}$$

分解と証明の関係(ならば, 前件)

分解



$$\begin{array}{c}
 \frac{\Gamma_1, \Gamma_2 \vdash \Delta, A \quad \frac{\Gamma_1, B, \Gamma_2 \vdash \Delta}{B, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (EL)}}{A \rightarrow B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta, \Delta} \text{ (}\rightarrow\text{L)} \\
 \frac{A \rightarrow B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta, \Delta}{A \rightarrow B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (CR)} \\
 \frac{A \rightarrow B, \Gamma_1, \Gamma_2, \Gamma_1, \Gamma_2 \vdash \Delta}{\Gamma_1, \Gamma_1, \Gamma_2, \Gamma_2, A \rightarrow B \vdash \Delta} \text{ (EL)} \\
 \frac{\Gamma_1, \Gamma_1, \Gamma_2, \Gamma_2, A \rightarrow B \vdash \Delta}{\Gamma_1, \Gamma_2, \Gamma_2, A \rightarrow B \vdash \Delta} \text{ (CL)} \\
 \frac{\Gamma_1, \Gamma_2, \Gamma_2, A \rightarrow B \vdash \Delta}{\Gamma_2, \Gamma_2, \Gamma_1, A \rightarrow B \vdash \Delta} \text{ (EL)} \\
 \frac{\Gamma_2, \Gamma_2, \Gamma_1, A \rightarrow B \vdash \Delta}{\Gamma_2, \Gamma_1, A \rightarrow B \vdash \Delta} \text{ (CL)} \\
 \frac{\Gamma_2, \Gamma_1, A \rightarrow B \vdash \Delta}{\Gamma_1, A \rightarrow B, \Gamma_2 \vdash \Delta} \text{ (EL)}
 \end{array}$$

推論規則

$$\frac{\Gamma_1 \vdash \Delta_1, A \quad B, \Gamma_2 \vdash \Delta_2}{A \rightarrow B, \Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2} \text{ (}\rightarrow\text{L)}$$

分解と証明の関係(否定, 後件)

分解

$$\begin{array}{c} \Gamma \vdash \Delta_1, \neg A, \Delta_2 \\ \downarrow \\ A, \Gamma \vdash \Delta_1, \Delta_2 \end{array}$$



$$\frac{\frac{A, \Gamma \vdash \Delta_1, \Delta_2}{\Gamma \vdash \Delta_1, \Delta_2, \neg A} (\neg R)}{\Gamma \vdash \Delta_1, \neg A, \Delta_2} (E\neg)$$

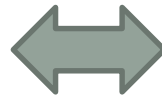
推論規則

$$\frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A} (\neg R)$$

分解と証明の関係(否定, 前件)

分解

$$\begin{array}{c} \Gamma_1, \neg A, \Gamma_2 \vdash \Delta \\ \downarrow \\ \Gamma_1, \Gamma_2 \vdash \Delta, A \end{array}$$



$$\frac{\frac{\Gamma_1, \Gamma_2 \vdash \Delta, A}{\neg A, \Gamma_1, \Gamma_2 \vdash \Delta} \text{ (}\neg\text{L)}}{\Gamma_1, \neg A, \Gamma_2 \vdash \Delta} \text{ (EL)}$$

推論規則

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta} \text{ (}\neg\text{L)}$$