Understanding Dynamics on Network Structure

Applying New Visualization Method to Coupled Chaotic Systems

Kazeto Shimonishi,[*1] *Junya Hirose,*[*2] *and Takashi Iba*[*1]

**1 Faculty of Policy Management, Keio University*

**2 Faculty of Environment and Information Studies, Keio University*

Introduction

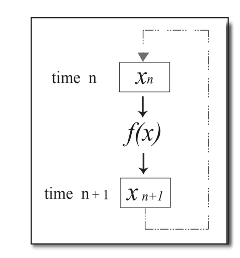
In this research, we apply new visualizing method, which we call “Footprints of Chaos", to coupled chaotic systems. In the method, the trajectory of nonlinear dynamics can be understood as a visual pattern on 2-dimentional plane, converting the value of function, like a logistic equation, into an angle. This method has an ability to express various patterns depending on the control parameter. Therefore it is useful to understand the shape of attractor because the generated figures show the different patterns according to the type of attractor, such as fixed, period, and chaotic region. We should emphasize that this method has an advantage to visualize the unstable periodicity in the region of the chaotic area, in which we can observe the mixture of periodic patterns and chaotic complexity.

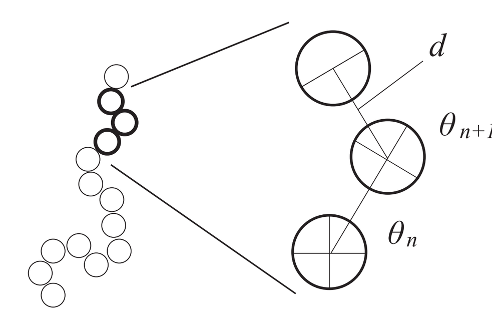
In this presentation, we propose the method to observe the both of a network structure and internal dynamics of each elements at the same time, and show some patterns on two types of network; the coupled map lattice (CML) and globally coupled map (GCM).

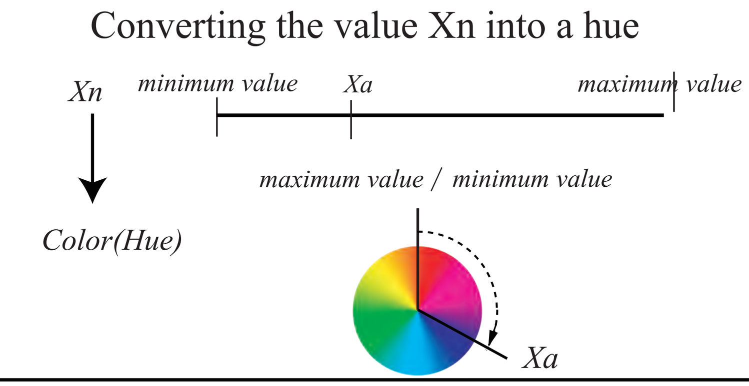
The Proposed Method “Footprints of Chaos”

$$x_{n+1} = f(x_n)$$

(1) Drawing a circle with radius r.
(2) Converting the initial value x_0 into the angle θ_0 and drawing a circle, which is also with the radius r on the point at the distance d and the direction θ_0 from the center of the previous circle.
(3) After calculating the value of x_1 with the equation, converting the value x_1 into the angle θ_1 and drawing the next circle as the same way.







Coupled Chaotic System

1.CML (Coupled Map Lattice)

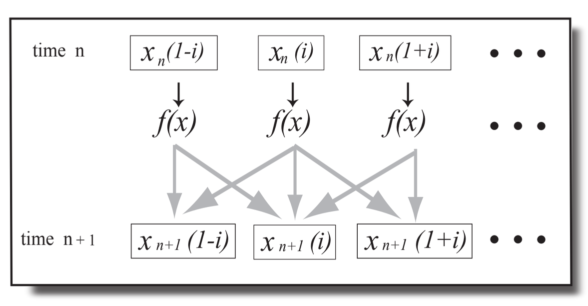
A CML is a dynamical system with discrete time, discrete space and every node is neighbors of one another and the chaotic elements interact with both neighbors function. At here, we prepare the chaotic elements as following function.

$$f(x_{n+1}) = 1 - a x_n^2$$

the dynamics is given by

$$x_{n+1}(i) = (1-e) f(x_n(i)) + \frac{e}{2}\left(f(x_n(i+1)) + f(x_n(i-1))\right)$$

i = lattice point (i = 1,2,⋯N = system size)
n = time step

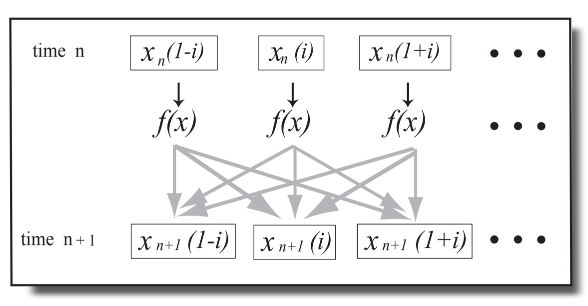


The schematic figure of interaction in CML

2.GCM (Globally Coupled Map)

The GCM is a synamical system that every chaotic elements interact with each other, and the dynamics is given by

$$x_{n+1}(i) = (1-e) f(x_n(i)) + \frac{e}{N}\sum_{j=1}^{N} f(x_n(j))$$



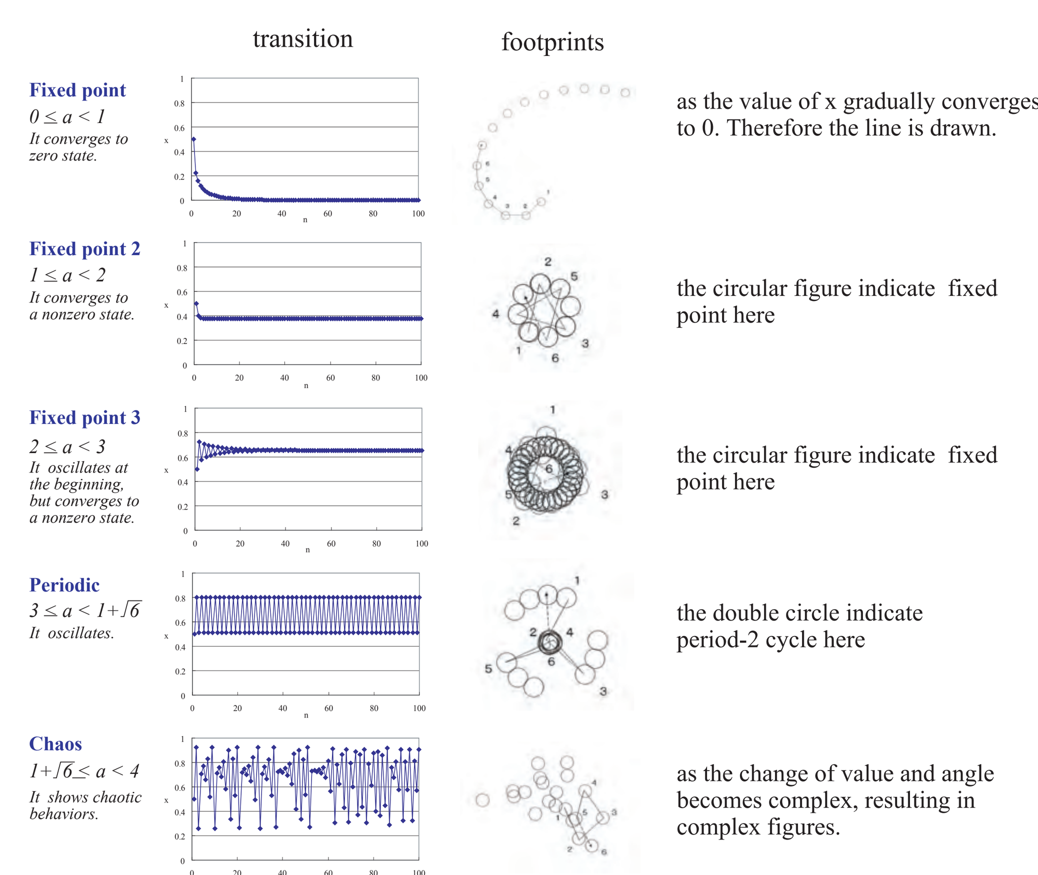
The schematic figure of interaction in GCM

Applying the method to logistic map

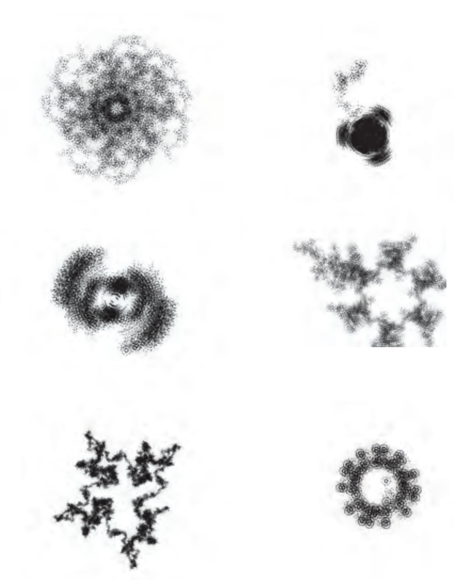
For instance, applying the method into the logistic map as follows to understand the method.The equation is given by

$$x_{n+1} = a x_n (1 - x_n)$$

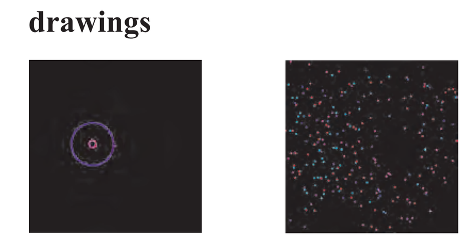
where $0 \leq x_n \leq 1$ and $0 \leq a \leq 4$.



Example of the Patterns with Changing Control Parameter ***a***

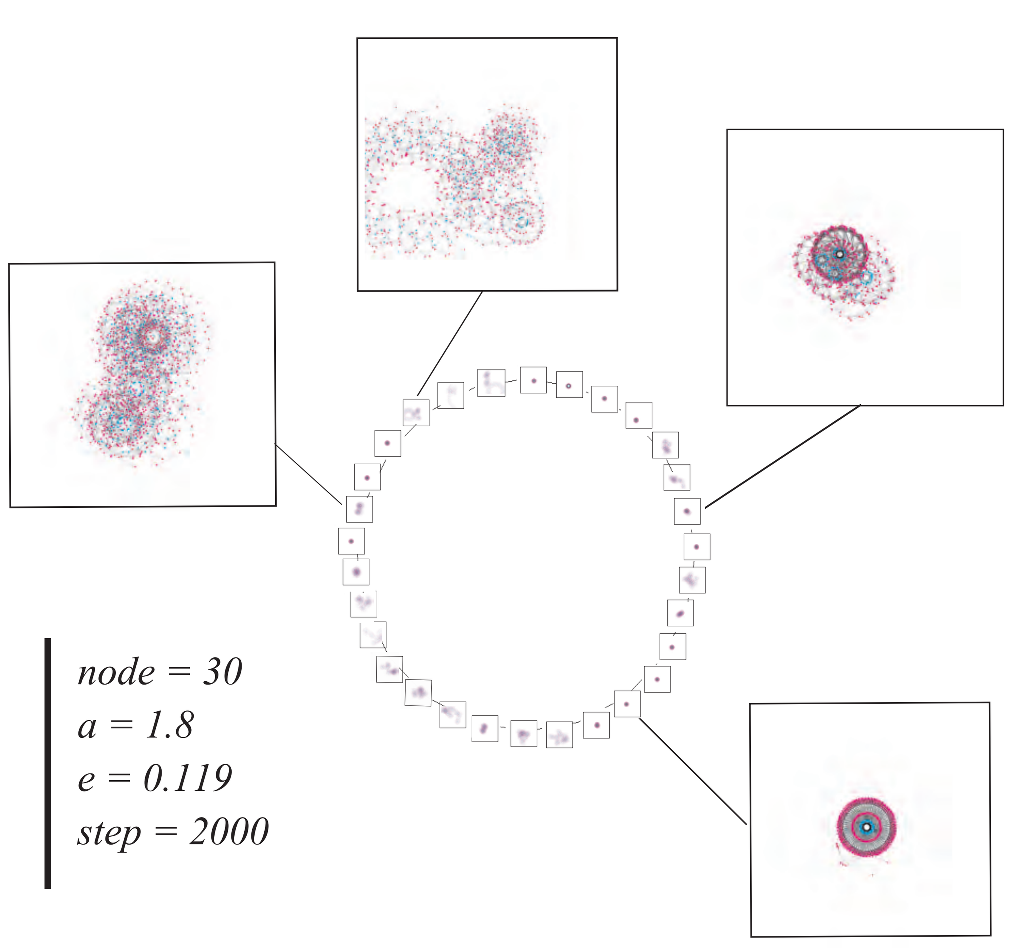


Example of the coloring Patterns of both double circle and chaotic drawings



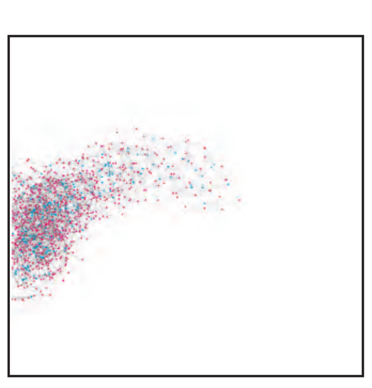
Applying the method to Coupled Chaotic System

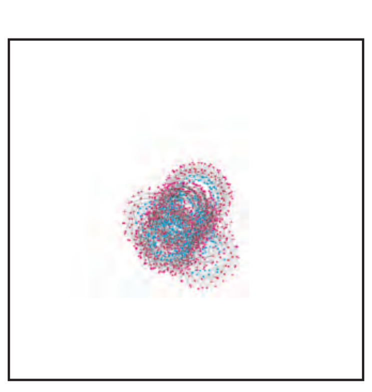
Footprints of Chaos on CML



node = 30
a = 1.8
e = 0.119
step = 2000

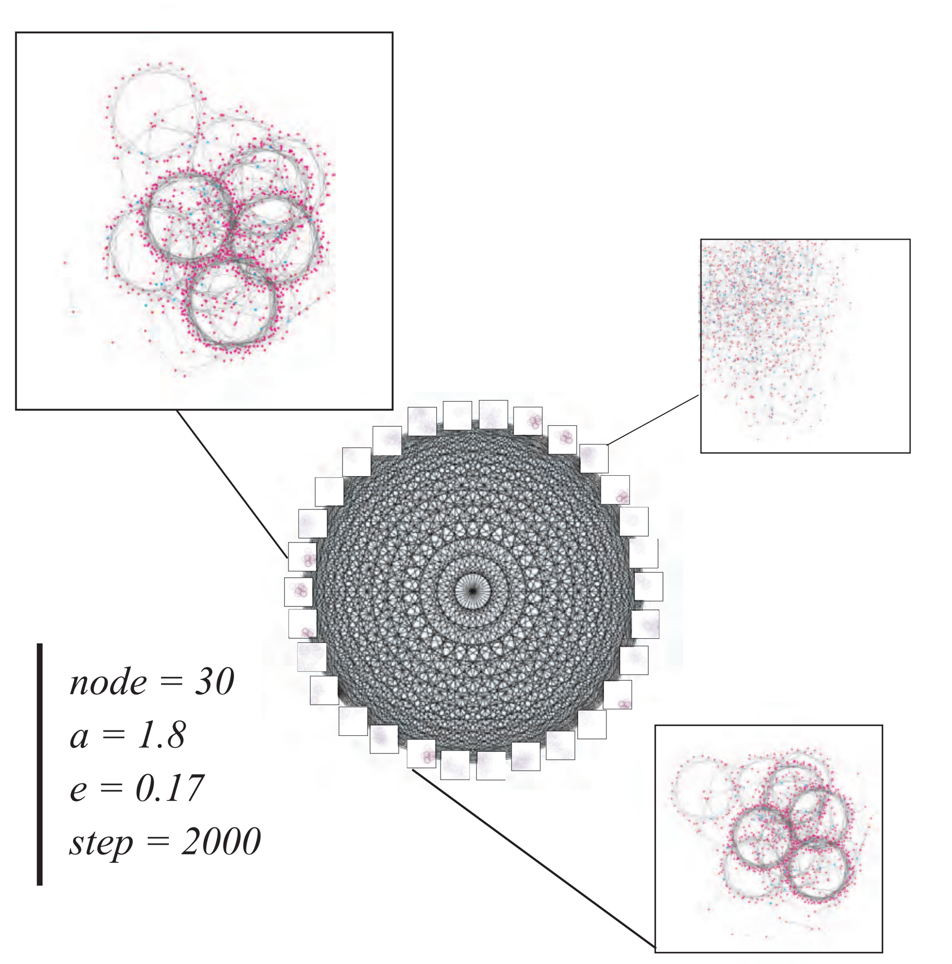
Other patterns under differnce condition on CML





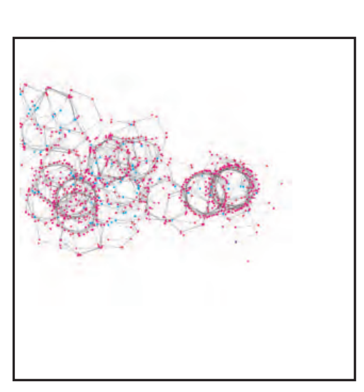
In CML, the phenomenon called STI(Spatiotemporal Intermitency) is a one of the charactaristic.As the nonlinearity a is increased further, the domain is no longer stable and the pattern starts to collaspe.Each lattice point alternates irregularly between the orderd states with patterns and spatially disorganaized,temporally chaotic regions.
We could find the features with propoing method as shown in above.The observed figures are sometime drawn as a circle and sometime chaotic.

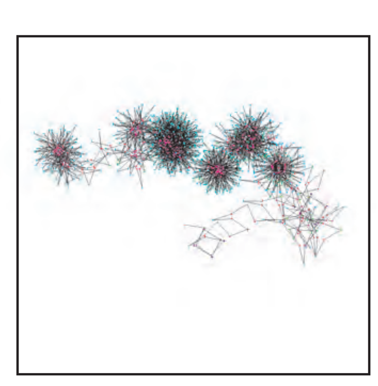
Footprints of Chaos on GCM



node = 30
a = 1.8
e = 0.17
step = 2000

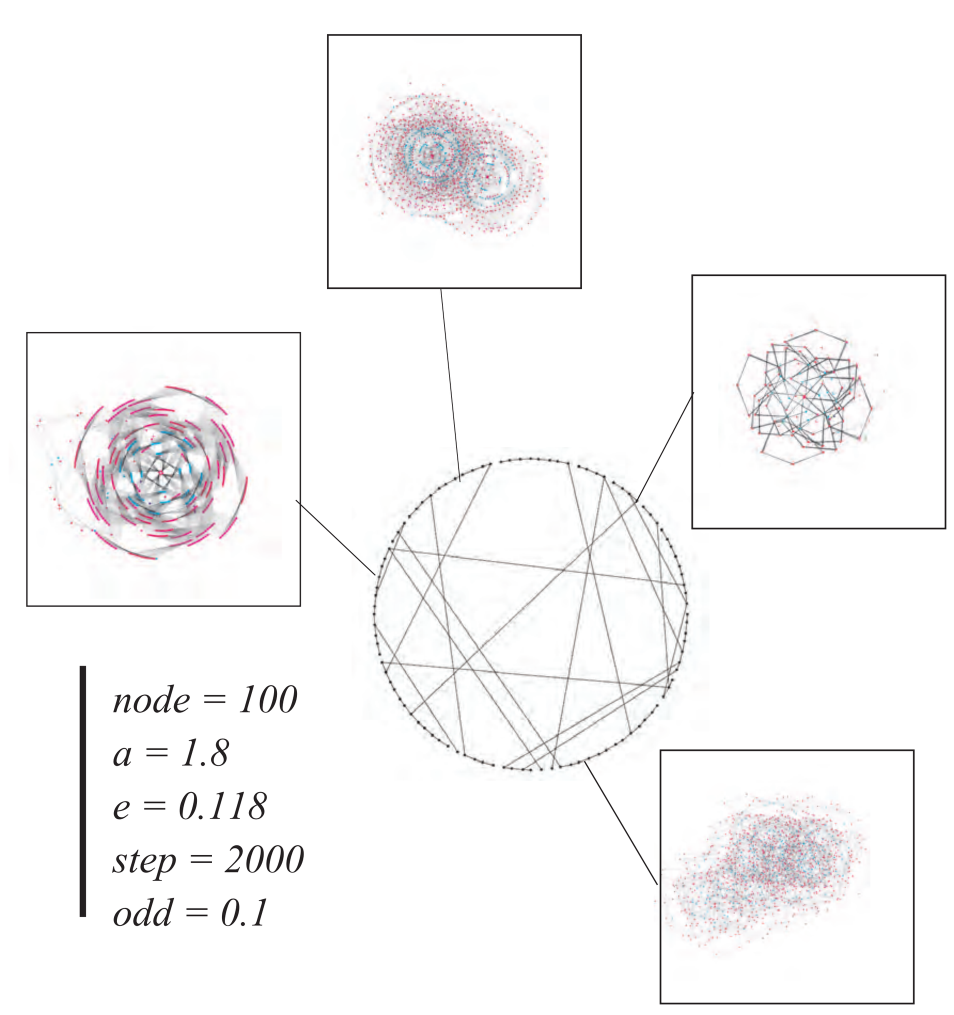
Other patterns under differnce condition on GCM





In GCM, there are “Partially Orderd area” . At here,many states with different clusterings coexist as attractors. The features are observed with propoing method as shown in above.We could find some attractor coexist in the same node.In the same network,some elements of node behave chaotic but some elements have attractors in the same node.

Footprints of Chaos on Small World Network



node = 100
a = 1.8
e = 0.118
step = 2000
odd = 0.1

At here, we are try to visualizing the method applying to the Small World Network. Not as like CML and GCM, there are seldom reserch applying the coupled chaotic system to the Small World Network. However, we could find the interesting results as show in above. Not only finding figures as same as CML and GCM but also another patterns.

Therefore, we would like to observe and analyze the behavior of each elements to find the new patterns or some features in future work.

References

- *K.kaneko ed., Theory and Applicatons of Coupled Map Lattice,Wiley, 1993*
- K.kaneko and I.Tuda, *Complex Sysytem: Chaos and Beyond*, Springer,2000
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