The Footprints of Chaos

A Novel Method and Demonstration for Generating Various Patterns from Chaos

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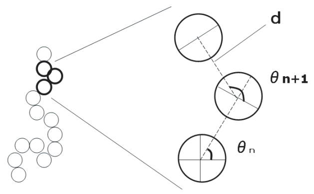
1. Introduction

We propose a new method for generating various patterns from chaos, which visualize the history by converting the value into an angle and plotting the circle on the two-dimensional plane. Chaos is one of the dynamical phenomena, which the behavior seems irregular but acts under a deterministic rule. Interestingly, the behavior of chaos is not only irregular, but also have a complex structure in the system. We show the map and movie to demonstrate that the proposed method is able to generates a lot of —— an infinite of —— various patterns according to the initial values and a control parameter.

2. The Proposed Method

The proposed method is to visualize the nonlinear dynamics by converting the value into an angle and plotting it on the two-dimensional plane, as a following figure. In this demonstration we apply the method into the logistic equation $x_{n+1} = a\, x_n(1 - x_n)$, which was proposed by the biologist R.M. May. The equation is known as the model of the relation between certain species' population of the parental generation and next generation. Note that the variable x_n represents the population of n-th generation ($0 \le x \le 1$), and the parameter a represents intrinsic growth rate ($0 \le a \le 4$), which is called "control parameter".



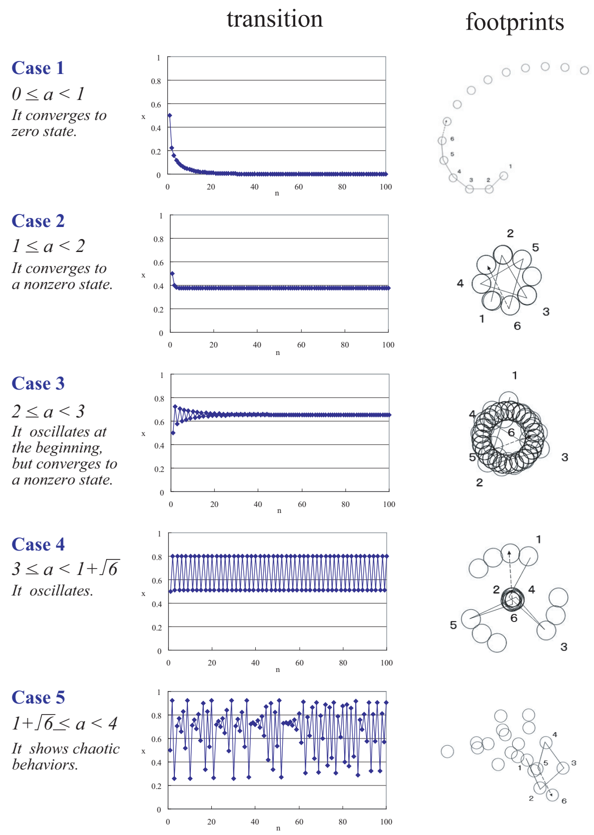
(1) Drawing a circle with radius r.

(2) Converting the initial value x_0 into the angle θ_0 and drawing a circle, which is also with the radius r on the point at the distance d and the direction θ_0 from the center of the previous circle.

(3) After calculating the value of x_1 with the equation, converting the value x_1 into the angle θ_1 and drawing the next circle as the same way.

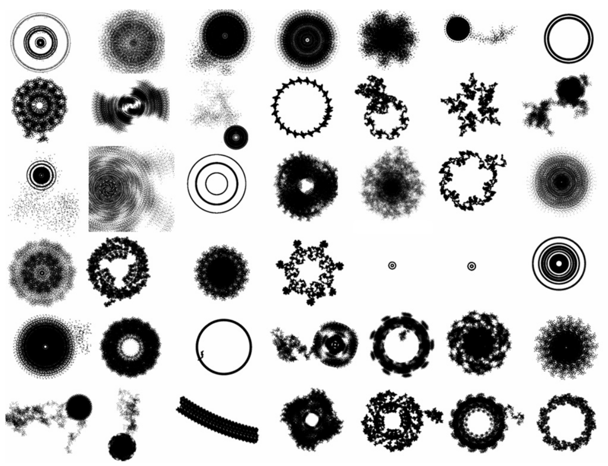
(4) Repeating the same procedure for x_n.

The function has been received attention because the simple nonlinear function can generate the complex transition of the value. Here, we introduce the features of patterns generated by the proposed method. In **case 1**, the angle decreases down to 0, as the value of x gradually converges to 0, therefore the line goes over the border of the canvas. In **case 2**, the angle is getting fixed, since the value of x converges to the fixed point, therefore the circular figure is drawn on the canvas. In **case 3**, the value sneaks around due to the oscillation at the beginning, then the angle is getting fixed, since the value of x converges to the fixed point, therefore the circular figure is drawn on the canvas. In **case 4**, it oscillates between a larger value in one generation and a smaller value in the next. The two-layer structures are drawn, because the two angles are generated by the period-2 cycle. In **case 5**, it shows irregular and complex behavior. It is "chaotic" behavior to generate the different value forever, therefore it never reaches fixed point or periodic cycle. The pattern is much different from patterns in the case of fixed point and periodic cycle.



3. Overview of Footprints' Patterns

We show the footprints' patterns changes by changing both of the control parameter a and the initial value x_0 as a visual map and a movie. The following figure is samples of the patterns generated by the proposed method. The method is able to generate an infinite of various patterns in case of chaotic areas.



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