Perfect Matching

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Abstract

This is pseudocode for Perfect Matching in Bipartite Graphs, adapted from the paperback edition of *A Gentle Introduction to Optimization*, B. Guerin *et al.* This is used as a subroutine to find any perfect matching on a subgraph of the larger bipartite graph in the Hungarian Algorithm, whose goal is to find a minimum weight perfect matching.

The operator Δ is essentially an exclusive OR (XOR) of the two sets:

$$M \Delta P = (M \cup P) \setminus (M \cap P). \tag{1}$$

Executed on an alternating paths of links included in versus excluded from the set and a second path including at leaast part of that first set, it has the behavior of flipping the ones included versus excluded.

I believe this algorithm as stated assumes that the input graph H is connected.

Algorithm 3.4 Perfect Matching

```
Input  : Bipartite graph H = (V, E) with bipartition U,W where
         |U| = |W| ≥ 1
Output: A perfect matching M, or a deficient set B ⊆ U.
M := ∅
T := ({r}, ∅) where r ∈ U is any M-exposed vertex
while (1) do
{
    if ∃ uv where u ∈ B(T) and v ∉ V(T) then
    {
        if v is M-exposed then
        {
            P := T_ru ∪ {uv}
            M := MΔP
            if M is a perfect matching then { stop; }
            T := ({r}, ∅) where r ∈ ∪ is any M-exposed vertex.
        }
        else
        {
            Let w ∈ V where vw ∈ M
            T := (V(T) ∪ {v, w}, E(T) ∪ {uv, vw})
        }
    }
    else
    {
        stop B(T) ⊆ U is a deficient set
    }
}
```