Simplex Algorithm

May 17, 2017

Abstract

This is pseudocode for the core of the Simplex Algorithm, adapted from *A Gentle Introduction to Optimization*.

Algorithm 2.1 Simplex Algorithm

Input : Linear program (P) and feasible basis B
Output: An optimal solution for (P) or a certificate proving that (P) is unbounded

Rewrite (P) so that it is in canonical form for the basis B
Let $\vec{x}$ be the basic feasible solution for B
if $\vec{c_N} \leq \vec{0}$ **then**
$\{$
    stop
    ($\vec{x}$ is optimal)
$\}$
Select $k \in N$ such that $c_k > 0$
if $A_k \leq \vec{0}$ **then**
$\{$
    stop
    ((P) is unbounded)
$\}$
Let *r* be any index *i* where the following minimum is attained:

$$t = \min\left\{\frac{b_i}{A_{i,k}} : A_{i,k} > 0\right\} \tag{1}$$

Let ι be the r^{th} basis element
Set $B := B \cup \{k\} \backslash \{\iota\}$
Go to step 1

Notes:

1. Recall $\vec{c}$ is the vector of coefficients in our objective function $z(\vec{x})$, and c_k is the kth element of $\vec{c}$.

2. B is the set of columns in A that comprise our basis; N is the rest of the columns.

3. $\vec{c_N}$ is $\vec{c}$ with the columns corresponding to basis B removed.

4. A is the $m \times n$ matrix with linearly independent rows that comprise our constraints. A_k is the kth column of A (a vector, though we aren't writing it with the arrow above), and A_B or A_N is a matrix comprised of a subset of the columns of A, keeping them in the original order.